Thermal Acoustic Waves from Wall with Temporal Temperature Change

G Sakaguchi[a] ,M Tsukamoto[b] and A Sakurai[b]

[a]Tokyo Metropolitan University)
Asahigaoka 6-6 Hino City, Tokyo 191-0065,Japan
[b]Tokyo Denki Universityt
Kanda Tokyo 101-8457, Japan

Abstract. Although phenomenon of thermo-acoustic wave has been known for many years in some familiar experiences such as “singing flame” from Bunsen burner, recent trends of utilizing it for the industrial applications urge the understandings of basic details of the phenomenon itself. Here we consider, in this connection, the problem of acoustic wave generation from a particular heat source of solid wall whose temperature changes with time and the phenomenon of temperature change by standing wave oscillating in closed tube. For these we set a hollow tube whose temperature at its one end wall changes with time, and compute flow field inside using the molecular kinetic model, which is found to be more convenient for the boundary value fitting than the ordinary acoustic theory system to this problem. In practice, we use the Boltzmann equation with the BGK approximation, and compute two cases above in monotonic and sinusoidal temperature changes with time. Results of both cases show propagating density wave from the wall almost in acoustic velocity to the first case and the temperature decreases in average to the second case.

Keywords: Thermo-acoustics, Kinetic theory
PACS: 47.45.Ab, 51.10.+y

INTRODUCTION

The phenomenon of thermal-acoustics has been known for years [1] in some familiar everyday experiences, representatively as “singing flame” from Bunsen burner or thunderclap. In fact, sound wave propagation itself is given by the adiabatic change in air. Beyond these mere observations of the phenomenon, there have been various efforts of utilizing the effects for industrial applications. For example, the one for the refrigeration utilizing the effect of cooling material by acoustic wave [2], or Sterling engine by thermo-acoustic self-excited oscillation [3]. These trends are naturally stimulating the study for the basic details of the phenomenon itself. Here we consider the problem of thermal acoustic wave generation from a solid wall surface whose temperature changes with time and the problem of acoustic wave cooling of materials being effective for refrigeration as quoted in above. We approach these theoretically by the use of the molecular kinetic theory model [4] Notice that the traditional gas dynamics approach [5] based on the wave equation having a source term of entropy change with time inside gas flow is found to be not convenient to the present case of source at wall in boundary value fitting., while it is naturally incorporated in the kinetic model approach for the surface condition such as diffuse reflection at solid surface.

In practice, we compute one-dimensional and two flow fields inside a hollow tube in using the Boltzmann-BGK equation in a reduced form [5] with the diffuse reflection at walls for both cases of changing wall temperature with time and standing wave oscillating inside the tube. Results show propagating density wave in acoustic velocity to the first case and some decrease in the temperature in its mean value to the second case.

THE BOLTZMANN-BGK EQUATION

We have the Boltzmann-BGK equation for the molecular velocity distribution function $f(\mathbf{c},\mathbf{x},t)$

to the molecular velocity $\boldsymbol{c} = (c_x, c_y, c_z)$, the spatial coordinates **x**=(*x, y, z*) and the time t as,

$$\frac{\partial f}{\partial t} + c\frac{\partial f}{\partial x} = \nu(f_0 - f). \tag{1}$$

where $\nu = \nu(x,t)$ is the collision frequency and $f_0 = f_0(\mathbf{c}, \mathbf{x}, t)$ is given by the local Maxwellian as

$$f_0 = \frac{N}{(2\pi RT)^{3/2}} e^{-C^2/(2RT)}$$

with R being the gas constant and

$$\mathbf{C} = \mathbf{c} - u, C = |\boldsymbol{C}|,\ N = N(x,t) = \int f d\boldsymbol{c}, u = u(x,t) = N^{-1}\int \boldsymbol{c} f d\boldsymbol{c},$$

$$T = T(x,t) = (3NR)^{-1}\int C^2 f d\boldsymbol{c},\ p = NRT$$

The collision frequency ν can be expressed as

$$\nu = k_0 N / K_n,$$

where $k_0 = 8/5\sqrt{\pi}$, K_n is the Knudsen number given by l/L, with l the mean free path and L the representative length.

For the convenience of numerical computation of 2D or 1D flow, we use the reduced distribution functions g, h and their local Maxwellians g_0, h_0, which are given by

$$g = \int f dc_z,\ h = \int C_z^2 f dc_z;\ g_0 = (N/\pi T)\exp.\{-(C_x^2 + C_y^2)/T\}, h_0 = 1/2.T g_0 \quad , \quad \text{for 2D}$$

$$g = \iint f dc_y dc_z, h = \iint (C_y^2 + C_z^2) f dc_y dc_z; g_0 = N(\pi T)^{-1/2}\exp(-C_x^2/T), h_0 = T g_0 \quad \text{for 1D}$$

the integral form being affected along the characteristics to the small time step Δt

$$,(g,h)(\boldsymbol{c}, \boldsymbol{x} + \boldsymbol{c}\Delta t, t + \Delta t) = (g,h)(\boldsymbol{c}, \boldsymbol{x}, t) + \Delta t[(g_0, h_0) - (g,h)](\boldsymbol{c}, \boldsymbol{x}, t) \qquad (2).$$

in the non-dimensional form based on the reference density N_0, temperature T_0 or the most probable velocity $\xi_0 = \sqrt{2RT_0}$ and length L.

BOUNDARY CONDITION AT WALL SURFACE

We use the diffuse reflection condition at the wall surface and set the function *f* at the wall to the local Maxwellian f_W to which its temperature is T_w, its velocity is zero and its density N_w is given by the condition of zero mass flux across the surface. It is given more explicitly by the condition :

$$\int_{c_n > 0} c_n f_W d\boldsymbol{c} + \int_{c_n < 0} c_n f\, d\boldsymbol{c} = 0 \tag{3}$$

at the boundary surface、where c_n is the normal component of the molecular velocity at the wall surface and f_W is given by the above Maxwellian as,

$$f_W = \frac{N_w}{(\pi T)^{3/2}}\exp\left(-\frac{V^2}{T}\right), \quad V^2 = V_x^2 + V_y^2 + V_z^2$$

to have,

$$N_w = -2\sqrt{\frac{\pi}{T_w}} \iint_{c_n<0} c_n g_0 dc_x dc_y \,.$$

and we have

$$g_W = \int f_W dc_z = \frac{N_W}{\pi T_W} \exp.\left(-\frac{C_x^2 + C_y^2}{T_W} \right) \text{ for } c_n > 0 \text{ and } h_W = (1/2) T_W g_w$$

and the boundary condition:

$$g = g_0,\ h = (1/2) T_w g_0 \,;\quad g = g_0, h = T_w g_0 \qquad (4)$$

at the boundary wall surface respectively for 2D and 1D cases.

The condition of the diffuse reflection models the interaction of gas molecules with the molecular structure of the solid, in contrast to the macroscopic gas dynamic boundary condition which is not related to the internal structure of the wall material and not suitable to incorporate the interaction process.

COMPUTATION OF THERMAL ACOUSTIC WAVE IN TUBE

We consider thermal acoustic wave from a particular heat source of solid wall surface whose temperature changes with time, its propagation and its effect on boundary wall..We set a hollow tube as shown in Fig.1, and compute flow field inside using the molecular kinetic equation of eq. (2) subject to the boundary condition of eq.(4), In practice, we compute the following two cases:

(i) Wave generation from a wall of temporal temperature change.
(ii) Temperature change by standing wave in closed tube.

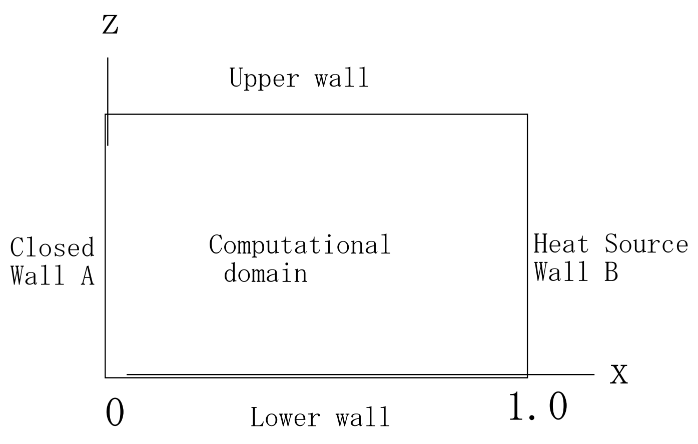


FIGURE 1. Configuration of computational domain

(i) Computation of generation and propagation of wave from solid wall with changing temperature *T* with time *t*, in one and two-dimensional(1D and 2D) cases.

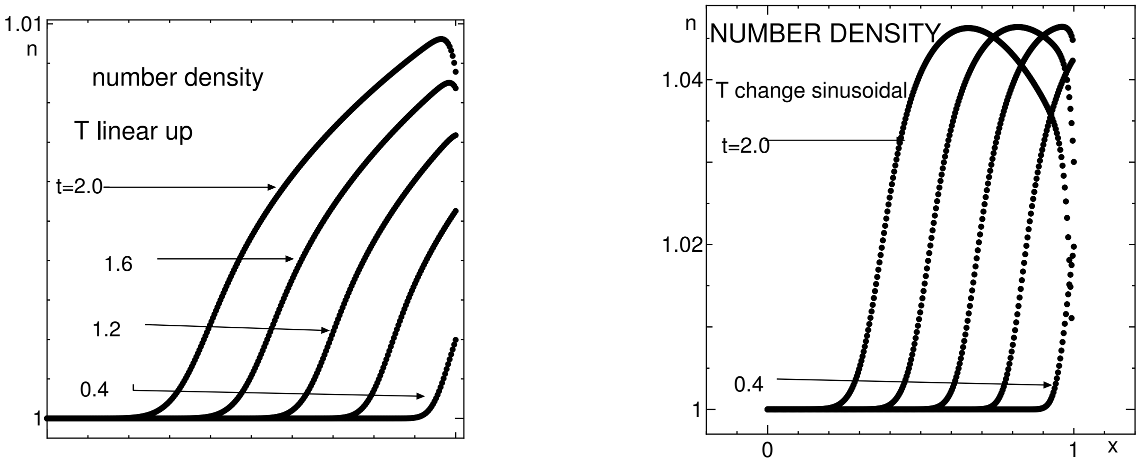
We set

$T = T_0 + f(t),\ f(t) = \alpha t$ or $\sin \omega t$

on the wall B surface at $x = 1.0\ (L)$

here T_0 is the initial temperature of the gas in tube and α, ω are constants..

Fig.2(a),(b) show examples of the results of one-dimensional computation of the density distribution respectively for the first and the second cases of $f(t)$, both show propagating density wave velocity $\tilde{c}$ in almost acoustic velocity as $\tilde{c} \approx 0.7\xi_0$ (a) and $\tilde{c} \approx 0.75\xi_0$ (b).



(a) (b)

FIGURE2. Propagating density wave from end wall of changing temperature with time:
(a) $f(t) = \alpha t$ (b) $f(t) = \sin \omega t$; $K_n = 0.0015,\ \alpha = 0.1,\ \omega = 10\pi$

Next we performed two-dimensional (2D) computation to the first case of $f(t) = \alpha t$ to see the effect of boundary on the propagation characteristics of the wave. We used the diffuse reflection condition to all the surfaces of A,B, upper and lower walls. Results are shown in Fig.3 for an example of the wave propagation for $\alpha = 0.1$

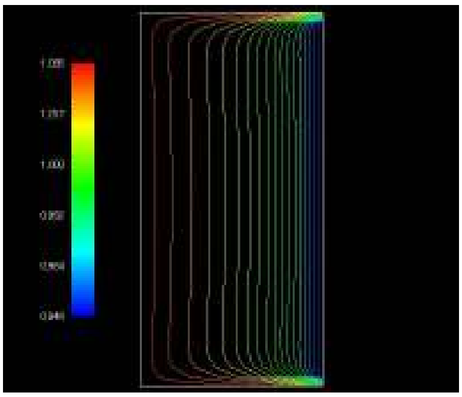
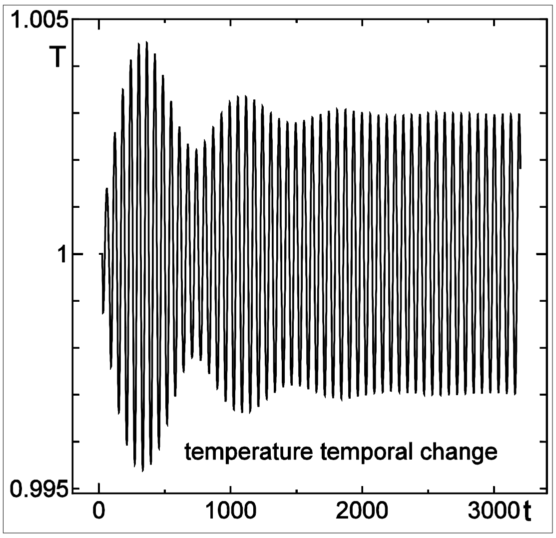


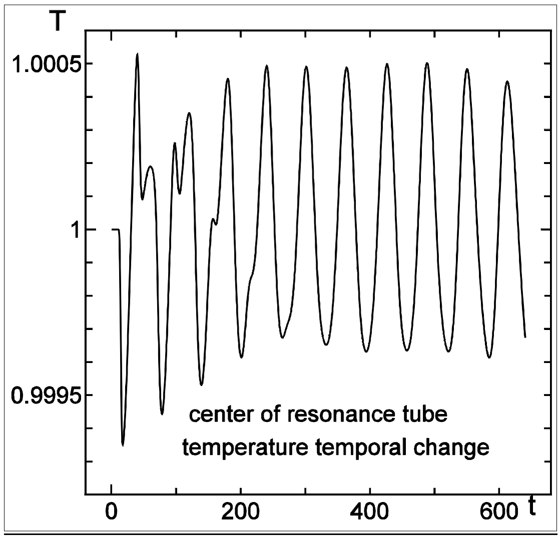
FIGURE3. Two-dimensional computation of the propagation of 2D density wave from end wall B of changing temperature with time as $f(t) = \alpha t$、α=0. 1

(ii) Effect of stationary wave on the temperature.

We produce 1D standing wave in tube by oscillating the temperature of surface wall B at x=1 as $\sin \omega t$ above and examine the temperature of gas at the at x=0 and x=0.5 . Fig.4 shows the changing feature of the temperature T_B at $x = 0$ (a) and $x = 0.5$ (b). We can notice a tendency of the temperature decreasing with time in its average as been shown in famous paper of reference (2).



(a) at $x = 0.0$



(b) at $x = 0.5$

FIGURE 4. Temperature (a) $T_B(t)$ at x=0 (b) $T_B(t)$ at x=0.5

SUMMARY

Computed thermal acoustic waves in hollow tube utilizing the molecular kinetic model of the Boltzmann-

BGK equation with the diffuse reflection condition at a wall..

Observed generation of acoustic wave from solid wall which changing temperature with time, iin 1D and 2D computations..

Noticed a tendency of temperature decrease with time in average in 1D computation of stationary wave by oscillating the wall temperature of the surface wall.

REFERENCES

1. Lord Rayleigh: The theory of sound, Vol. II (1945).
2. P. Merkli and H.Thomann: J.Fluid Mech. **70** 161-177(1975).
3. S. Backhaus and G. W. Swift: Nature **399** (1999)335.
4. A.Sakurai and F. Takayama: Phys. Fluids, **15**,1282-1294 (2003).
5. N. Sugimoto, D. Shimazu and Y. Kimura: Phys. Fluids, **20**, p.024103 (2008).

.